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IMPRECISION AND EXPLANATION*

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The paper,[1] analyses the role of measurable concepts in deductive explanation. It is shown that such concepts are, although imprecise in a defined sense, exact in that neutral candidates to them do not arise. An analysis is given of the way in which imprecision is related to generalisation, and it is shown how imprecise concepts are incorporated in testable deductive explanations.

1. Imprecision and inexactness. In [8], the problems of theoretical explanation posed by the inexactness of scientific concepts were analyzed. In this paper, the special case is considered of quantities, measurable concepts that are exact but imprecise, and their role in deductive explanation is more fully discussed.

All the dilemmas of definition of inexact concepts arise because being a positive instance of a concept in an all-or-nothing thing. A neutral candidate to the concept 'tributary', e.g. a river branch which is the shorter but has the greater volume flow ([8], section 3), cannot be accommodated by describing it as 'half a tributary'. Similarly, one of Lakatos' counter examples [6], ([8], section 6) cannot be disposed of as '*n* per cent polyhedral'. The application of deductive logic requires a clear-cut assignment.

But there is one important class of scientific concepts, namely those measurable on at least an interval scale (Stevens, [12]; Ellis [4], chapter 4) which do not pose these assignment problems. The empirical meaning of the term denoting such a concept, as displayed in its diversity of correct uses, and in particular those features, discussed in [8], which normally give rise to inexactness, can be accounted for by ascribing an *imprecision* to the concept which yet remains exact.

This may be illustrated by another example of Quine's ([10], chapter 4), which he lumps together with 'tributary' as an instance of a vague (i.e. inexact) concept, but which should really be placed in sharp contrast with it. His example is the concept 'the size of a city'. As with 'tributary', the problem arises because, for the concept to be used to convey information, it must occur in at least two independent

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law statements ([8] section 3; [7] section 2). For example, the size of a city might be characterised by the number of people travelling into a common centre to work, or by the number of households paying rates to one authority, or by postal areas, or by telephone exchanges. Conversely, given the size of a city, one expects to be able to infer something about all these things.

But again, there will obviously be discrepancies between the values of the size of a given city measured by these different criteria. Given any particular value for the size of a city, some criteria applied to one city will perhaps fit it, but certainly most will not, so that almost any city will be at best a neutral candidate for a particular value. It seems exceedingly likely that *no* value for the size of a city has any positive instances at all, and even those values with neutral candidates are thinly scattered among the mass of those values with only negative instances.

It seems clear, therefore, that an analysis in terms of inexactness applied to particular values of such a concept does not exhibit at all well the normal correct use of the term denoting it. It is not senseless to say that the size of Cambridge, for example, is 93,000, even though this value is not sensibly to be distinguished from 93,001 or 93,010.

Starting as before from the law cluster of accepted statements ([8] section 3) about the size of Cambridge, a more plausible analysis may be attempted. Each accepted statement about the size of Cambridge may be used as suggested above to determine a value of the size of the city. The set of values so obtained will be included in some interval of values. Let the shortest such interval be *I*, e.g. [92,000, 94,800]. Then we may say: 'the size of Cambridge is in *I*', provided this is *not* taken to mean that there is one precise city size somewhere inside this interval which we could find but for the contrariness of the city council, post office, and business community. Given that the concept is determined by this variety of accepted statements, there just *is no* such precise size. What 'the size of Cambridge is in *I*' means is that, for each accepted statement about city sizes, there is a value in *I* for which that statement is true of Cambridge. The interval *I* in this sense represents that property of Cambridge which is correctly termed its 'size'. The length of *I* gives what I propose to call the '*imprecision*' in the concept. (Chwistek, [2], p. 256).

It is clear that imprecision in such a concept expresses that aspect of it which is normally responsible for inexactness, since the interval of imprecision has to contain values derived from any member of the law-cluster of statements defining the correct use of the term 'city size'. The point of the requirement stated above that an imprecise concept must be measurable on at least an interval scale should now be clear enough. To ascribe imprecision is to ascribe some interval of values. If the concept is measurable only on an ordinal scale, so that intervals of values are not defined, then imprecision cannot be ascribed to it, and it must remain inexact. For example, suppose that 'hardness' is only measurable on an ordinal scale, so that it is meaningful to assert that *A* is harder than *B*, but not that *A* is harder than *B* by *n* units of hardness. Then the discrepancies that may arise from applying different criteria (methods of measurement) on some of which *A* appears harder than *B*, and on others of which *B* appears harder than *A*, cannot be resolved by ascribing to *A* and *B* intervals of values of hardness which overlap. The concept remains inexact because imprecision cannot be ascribed to it, since one thing can no more be "*n* per cent harder" than another thing than it can be "*n* per cent polyhedral."

The role which the development of interval scales of measurement plays in transforming inexact into exact but imprecise concepts has not been sufficiently realised. (It should be noted at this point that the concept need not be capable of taking on any values in a continuous range, as the example of 'city size' shows. The restriction of the discussion in [7] to continuous variables ([7], p. 106) is quite unnecessary.)

Now the meaning of such a term as 'city size' is often a function of context. Words are not always used in their most general sense. One might, for instance, talk of 'the size of Cambridge' where it is clear from the context that only area, or number of voters, is intended, and traffic problems and telephones are not in question. This would be to limit the correct use of the term in that context, to invoke a restricted form of the concept limiting the inferences that may properly be drawn from the statement about it. The point of making such a restriction will normally be that the discrepancies between the members of this smaller sub-set of statements, selected from the law cluster, are much less than those between all the statements in the cluster. Hence in a context in which the form of a concept is thus restricted, its imprecision may be much less than that of its most general form.

It appears then, that imprecision in a concept is a function of the context in which it is invoked. The point of this admission is that this is not to say that the imprecision of a concept invoked in a variety of contexts is indeterminate or vague or inexact. The imprecision may be quite clearly defined as a function of context. A term cannot after all be used simultaneously in two different contexts, and any uncertainty in the imprecision may be ascribed to uncertainty about the context. This point is made because it is essential to the argument that a concept which is imprecise, although it accommodates those features of experience which normally make for inexactness, is nevertheless itself exact. The question of neutral candidates simply does not arise in the same way.

The following objection might be raised to this view. It has been assumed so far that applying any single criterion of a city size yields a precise value. In practice, of course, what is obtained is itself an imprecise estimate. Now suppose this estimate straddles the end-point of the postulated interval of imprecision. Does this not require an arbitrary decision to assign it as either a positive or negative instance; is such an estimate not a neutral candidate? In other words, is it not just as much an idealisation to set precise limits to an interval of imprecision as to give a single precise value to the original concept? If so, is there not an endless regress, with intervals of "second-order" imprecision for the end-points of the first interval, and so on? Might it not be that no finite number of higher-order imprecisions could fully capture the empirical vagueness of the concept?

Part of the answer to such an objection is that it has not been made sufficiently clear just what is meant by a value or by one interval of values being contained in another. One may specify, say, that an interval is taken to be a closed interval and that by 'contained' is meant 'entirely contained', so that any estimate or reading straddling the end point is a negative instance. This is not an arbitrary assignment rule, since it does not make any reference to, or single out, any particular method of measurement or set of statements about the concept. It is a general clarification of what is understood by an interval of imprecision: a convention which can, and should, be agreed on as applying to all measurable concepts.

The rest of the answer to the objection is that the end points of the interval can be made precise because the interval of imprecision can always be extended to cover all but a very small proportion of the most doubtful cases: those arising from the most trivial, least analytic, least confirmed statements in the law cluster. Setting aside the question, previously considered, of the context dependence of the *shortest* interval of imprecision that will cover the discrepant results of using all the statements about the concept accepted in that context, it is pertinent to note that an interval of imprecision can always be lengthened without falsifying any of the accepted statements. If a city size is in I, then it is in any longer interval, I^*, containing I.

In this second point, it needs to be made clear that the issue has not been evaded by referring to "all but a very small proportion of the most doubtful cases." It has already been observed [8] that any scientist is prepared to ignore some small proportion of discrepant data. In [7] this point was too closely linked to a particular (Normal or Gaussian) error distribution, which many measurable concepts do not satisfy: the arguments advanced there in fact apply quite generally. The point is merely that a suitably small proportion of doubtful cases can simply be ignored, as arising from some mistake in observation and therefore not needing to be accommodated within the interval of imprecision.

For this variety of reasons, it seems to me legitimate to stop at the first order of imprecision, i.e. to set precise limits to the interval of imprecision, and to take imprecise concepts to be exact.

2. Sources of imprecision. Having introduced the notion of an imprecise concept, it seems better to continue the analysis in terms of a more significant concept than that of 'the size of a city', namely that of 'length'. (Parts of the ensuing argument have been given in [7], but it is more convenient to repeat them here in a wider context than to make continual cross-references.) 'Length' is a significant scientific concept just because its law cluster has so many members, by virtue of which such a variety of inferences can be drawn from a statement of length. Conversely, any of these laws may be invoked in a measurement of length: a length may be determined, for example, from the period of a pendulum, the rate of flow of a viscous fluid, the extension of a rod on heating or under stress, by sundry optical and mechanical means. Thus giving a value for the length of an object implies that any of these methods of measurement which could be applied to the object would yield this value. But equally, such diverse measures of the length will not correlate precisely (in general, not to more than a very few significant figures), so that a sufficient interval of imprecision must be provided to accommodate all the different readings.

Here we may anticipate an operationalist objection, to the effect that there is after all a perfectly good, unique, operational definition of length, and that any imprecision arising reflects limitations in the technique of measurement rather than in the precision of the concept measured. (Such limitations may, of course, range in theoretical importance from the wave nature of light to bad eyesight on the part of the observer, but such distinctions are not to the present purpose.) The two points made in the objection, about the uniqueness of the operational definition, and about the triviality of its imprecision, may be taken separately.

The fallacy inherent in any uniqueness requirement has in effect been exposed in [8]. To impose it is to assert that out of the law cluster of accepted statements about length, just one, invoked in the operational definition, is analytic, never to be given up, and all the rest are merely synthetic. Now, as was emphasised in [8] it is simply not true that just one statement about a scientific concept is taken to be permanently and solitarily analytic. Other difficulties in the operationalist view have already been sufficiently exposed in the literature and require little further comment. For example, if a different operation defines a different concept, so that celestial lengths are not lengths in a terrestial sense at all (Bridgman, [1], chapter 1, Dingle [3]), what is the justification for denoting them both by the same term 'length'? And then one must ask; how different in detail does an operation have to become before it generates a new concept? The main objection to such crude operationalism is precisely that it is incapable of accounting for the whole complex process of adjusting concepts and their exact forms which is the rationale of theoretical explanation.

The second point, that imprecision in measurement is trivial, incidental to the meaning of a term (Pap, [9], chapter 3, section B) is best answered by illustrating, in the case of 'length', the parallel context dependence of meaning and imprecision. A length measurement may be made for a very specific purpose, limiting the permissible inferences from it, and hence its imprecision. For instance, if an irregularly shaped table has to be fitted into a wall recess, the relevant length measurement is of an extreme value. If it has to be packed into a cylinder, the relevant length measurement is of a diagonal. If the measurement is to determine the surface area, and the table deviates appreciably from the rectangular, yet a third value is required. Each of these readings can be given much more precisely than in a statement of length which has to support all three inferences at once.

In this case, different methods of measurement are not, on the face of it, in question. One could, no doubt, say that finding the smallest recess or cylinder into which a table can be fitted constitutes a distinctive method of measurement, but it would be an unwarranted distortion of usage. In short, given common usage, it is just not true that a measurable concept and its imprecision are entirely determined by how it is measured, let alone by any uniquely privileged method of measurement.

It does not follow, of course, that no context dependence of meaning and imprecision is ascribable to changes in methods of measurement. It has already been remarked that the methods available for measuring a length depend on the value of the length. Similarly they depend on such properties as temperature and chemical composition of the object whose length is being measured. For example, it might be possible to measure the length of a copper rod by some very precise electrical method that could not be applied to a plastic rod. Then if inferences were confined to statements about copper rods (e.g. giving the coefficient of expansion of *copper*), one might be justified in reducing the imprecision in the length to what this method of measurement could achieve.

In fact, it seems clear that the precision of which a method of measurement is capable is very important in determining how analytic, how much a candidate for (operational) definitional status it is in the set of methods of measurement available in a given context. And it is just because this precision *is* a function of context that scientists are prepared to change, in a different context, to a different method of measurement. It would be absurd to let the precision of a concept be limited to that of its least precise method of measurement.

But equally, the choice between different methods is not hard and fast, and the grounds for saying that what all these different measures yield is a reading of a *length* is just that they are all warranted by law statements which are members of a common law-cluster; that where they can be applied together, their intervals of imprecision at least overlap. The point is merely that one very rarely uses the term 'length' in its full generality: we talk of the length of this or that object or substance, or over such a range of temperature or pressure, etc.

It should by now be clear that the sources of imprecision in a measurable concept include far more than just the imprecision, or experimental error, in one particular method of measurement. Yet it is this latter, often rather trivial, source that has received what little attention has been paid to "the approximate character of empirical knowledge" (e.g. Bridgman, [1], chapter 2). Moreover, because this source of imprecision is often ascribable to extraneous causes, or to the interaction of the observer with what he observes, imprecision has tended, with rare exceptions (Chwistek [2], p. 256), to be treated as quite incidental to the meaning of terms (Pap, [9], chapter 3, section B), a tiresome but trivial excrescence on the neat deductive structure of science. (Sellars, [11], p. 73; Hempel, [5], p. 101).

It needs emphasising that just as inexactness arises in non-measurable scientific concepts because their law cluster must contain at least two independent analytic statements ([8] section 4), so imprecision arises in measurable concepts because of the lack of precise correlation between the corresponding independent methods of measurement. This source of what I have called '*conceptual imprecision*' needs to be sharply distinguished from the more commonly discussed and more trivial *operational imprecision* that is ascribable to particular methods of measurement.

I have elsewhere given examples to illustrate the independence ([7], p. 110) of conceptual and operational imprecision. Another example involves the incubation period of a disease. Suppose that, for an individual *a*, the incubation period for disease *d* is operationally definable (from contact with a source of infection to the appearance of a rash) to within a few hours. This is the operational imprecision ascribable to that method of measuring the incubation period. But now lack of correlation with other equally firmly accepted measures (e.g. time to a characteristic rise in body temperature) may enforce a further, conceptual, imprecision (of say a day) on the concept '*a*'s incubation period for *d*' from which inferences are legitimately drawn about the appearance of either of these symptoms(rash or temperature rise). More generally, the point of measuring the incubation period will be to support inferences about the further development of the disease on the basis of law statements connecting these developments with the incubation period. Consequently, variation in the intervals between successive stages of the disease will impose further conceptual imprecision on '*a*'s incubation period for *d*' where statements about it are correctly used to predict the incidence of further symptoms.

I have supposed in this example that the conceptual imprecision is greater than the operational imprecision, but this need not be so. The different measures of *a*'s incubation period might all correlate to within the operational imprecision of any of them, and the further symptoms might all appear with similar punctuality. The point is that the sources of operational and conceptual imprecision are independent and in any given situation either may be dominant.

A further source of conceptual imprecision may be noted in the example, which will serve to introduce the discussion of the next section. Suppose that the incubation period for d in any individual a has a total imprecision of less than a day, but that it fluctuates between individuals by up to a week. Then there is a conceptual imprecision of a week in the more general concept, 'incubation period for d', invoked in inferences made about the progress of the disease in any, unspecified, individual. The relation of imprecision to generalisation illustrated by this example is elaborated in the next section, in which the source of imprecision in scientific concepts are further analysed to show how imprecise concepts can yet enter into testable deductive explanations. The analysis is a generalisation of the argument of [7] sections 3-5, to which reference should be made for technical detail.

3. Imprecision, generalisation and explanation. It has been asserted in the previous section that the imprecision of a concept is related to the generality of the context in which it is invoked. The assertion can best be clarified and supported by illustrations of the relation between imprecision and generality. The example has already been given of conceptual imprecision generated by generalising from 'a's incubation period for d' to 'incubation period for d'. This might be further extended if there were reason to refer to the incubation periods for diseases of a certain kind, of which d is one, where these varied from disease to disease by up to say two weeks. Then "incubation period for a disease of Kind K' would have conceptual imprecision of a fortnight. Similarly, in the example of 'the size of a city', one might pass from a statement about 'the size of Cambridge' to a statement about 'the size of county towns'. This generalising of the concept would require a further imprecision, due to the differences in size between different county towns, however measured. As already remarked, such a source of imprecision *could* be assimilated to that of discrepancies between methods of measurement by taking Cambridge and Oxford, say, to afford distinct measures of the size of county towns; as before, it seems an unwarranted and misleading distortion of usage. The notion that imprecision may be created by generalising a concept over individuals (e.g. individual county towns) seems quite clear and distinct from the notion that it may be created by generalising over methods of measurement.

Now the imprecision introduced by generalising over individuals may be so great to make the generalised concept virtually useless. 'Incubation period for a disease', for example, generalised over all diseases, is so imprecise as to make statements containing it almost completely uninformative. Similarly, the concept 'Age of man', generalised over men (i.e. humans, including women and children) of all ages, is useful only for saying, e.g., that men live longer than dogs and not as long as giant tortoises. Similarly again with such a concept as 'height of man'.

Such imprecision may be reduced by narrowing the classification of the individuals, especially by specifying a common value of a variable. So 'height of man aged 12' is much more precise than just 'height of man'. Now to make a list of heights for men of different ages is to state a *functional relation* between the age of men and their height. It is this that I take to characterise a *variable* concept, namely one that enters into a functional relation with another variable.

Generalising over values of a variable instead of over individuals is particularly valuable where individuals are not well defined, as with substances such as 'water', 'solid', 'chloride', 'rare earth', 'ideal gas'. The mass of an individual sample of water, for example, is closely proportional to its volume. 'Mass of water of volume V' is so much more precise than 'mass of water' generalised over individual samples of whatever volume that the latter is barely recognisable as an empirical concept at all. (There are, of course, a few named individuals: 'the mass of the Mediterranean' is a reasonably precise concept.)

The property of water expressed by this functional relation between its mass and its volume is represented by the characteristic parameter in that relation, namely 'density'. But now it is found that generalising over individual samples of water increases the imprecision even of 'density of water'. So again one looks for a functional relation with another variable, say temperature. The new, more precise, variable concept is now 'density of water at t° C'. The new relation generates yet another concept, 'coefficient of expansion', and imprecision in this may in turn bring in another variable, say pressure.

This, of course, is a logical, not a historical, account of how functional relations are arrived at. The point is that each successive functional relation deductively explains imprecision in the parameter of the previous one. At the lowest level, from the functional relation between mass and volume, the imprecision in the mass of an individual sample of water, whose volume is specified only within certain limits, is deducible. Similarly at the next level, the imprecision in the concept 'density of water at room temperature' is deducible from the functional relation between density and temperature, and the imprecision (ca. 10° C) in the concept 'room temperature'.

Thus one proceeds up the deductive hierarchy until one arrives at a functional relation whose parameters do not exhibit any conceptual imprecision, however much the operational imprecision is diminished by refinements of operational technique. These parameters consequently require no deductive explanation and are never conceived of as variables.

The need for such precise parameters at the top of the deductive structure becomes evident when one considers how imprecision percolates down it. For example, imprecision in its density increases the imprecision in the mass of a sample of water, and is in turn increased by imprecision in its coefficient of expansion. In fact, of course, the whole hierarchy of functional relations between pairs of variables can be replaced by one relation giving mass directly as a function of volume, temperature, pressure, etc. The intermediate derived variables can be dispensed with. Then to say that all the variables which affect the mass of a sample of water have been accounted for is to say that the parameters of *this* relation are precise. If we find that they are not, then another variable must be sought for, and hence a new functional relation with precise parameters, that will account for and thereby limit the imprecision. Without this limit, the deductive structure of functional relations would be untestable and uninformative, since with sufficient imprecision, a functional relation can be made to accommodate any data. (A more detailed technical development of these assertions is given in [7]).

Consequently the currently top level, ultimate, unexplained parameters of both statistical and deterministic theories must be conceived of as being precise, on pain of requiring further deductive explanation. The progress and point of measurement in science thus consists in replacing inexact by exact and imprecise concepts, and then in reducing and deductively limiting the imprecision in a theoretical structure, whose primitive, non-variable, concepts are taken to be both exact and precise.

4. Conclusion. It appears from this discussion that the admitted inadequacies of the straightforward deductivist account of theoretical explanation involving inexact concepts do not invalidate such an account where merely imprecise concepts are involved, provided that their imprecision is recognised and allowed for, and not dismissed as an unfortunate triviality. It is, I suspect, just their straightforward use in deductive explanation that motivates and justifies the development of quantitative concepts in science, and I hope that the above account will serve to make their rôle appear at once less mysterious and more interesting.

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