For *AJP*

Possibility, chance and necessity

1 Chancy possibilities

In this paper I defend the idea that *chances*, like the chances of radioactive atoms decaying in various times, embody possibilities of a contingent, positive and quantitative kind that I shall call *factual*. Something like this is certainly implicit in much that we casually say. If for example I say that my horse Pink Gin has *no* (i.e. zero) chance of winning the Derby, what I mean is that, as a matter of contingent fact, he *cannot* win. If I say that he has *a* (non-zero) chance of winning, I mean that in fact he *can* win. And this factual possibility, like that of an atom's decaying, is one which I take to come by degrees, of which chances provide a probability measure. That is, I take Pink Gin's chance of winning, like an atom's chance of decaying, to measure *how* possible those outcomes are, thus justifying, among other things, corresponding odds for bets on them.

If possibilities of this quantitative and apparently contingent kind really exist, they are obviously not just logical or physical. To deny that Pink Gin can win the Derby is not to say that the laws of logic or nature stop him winning. The possibilities that we take zero chances to rule out are as contingent as chances in general are. But they are also factual, in that we take it to be a matter of fact, and not merely of opinion, whether and to what extent it is possible for Pink Gin to win the Derby or for an atom to decay.

In calling non-zero chances 'possibilities', part of what I mean is that they are linked to corresponding necessities by the principle that, for all propositions 'H',

(a) H is possible if and only if (iff) not-H (~H) is not necessary.

This is why I think, for example, that Pink Gin can win the Derby (H) iff he is not bound to lose it (~H). So if the factual necessity of any contingent H is embodied in its chance – $ch(\mathrm{H})$ – being 1, and its impossibility in $ch(\mathrm{H})$ being zero, then because $ch(\mathrm{H})+ch(\sim\mathrm{H})=1$ for all H, we may express (a) as the condition that

(a′) $ch(\mathrm{H})>0$ iff $ch(\sim\mathrm{H})<1$.

This is actually not quite right, since a contingent H can fail to be impossible, not by there being a $ch(\mathrm{H})>0$ but by there being no $ch(\mathrm{H})$ at all, zero or otherwise. That is a perfectly good if negative way in which a contingent fact can be possible, and one we shall need to invoke later; but as what matters to start with is the positive way embodied in $ch(\mathrm{H})>0$, we may ignore it for the time being.

Part of what I mean by calling these contingent possibilities and necessities 'factual' is that – unlike their moral counterparts – they are linked to actuality by the so-called *axiom of necessity* ([1] ch. 2). This, given (a′)'s relation to (a), is best put by saying that

(b) nothing that is necessary can fail to be actual, and

(c) nothing that is impossible can be actual

(where the ‘can’ *is* logical). But not of course conversely: H can be factually possible without being actual, and actual without being factually necessary. Thus while no one will deny that (b) if Pink Gin must win he will win and (c) if he can’t win he won’t win, no one thinks that if he does win he had to win, or that if he can win it follows that he will win.

Expressed in terms of chances, (b) and (c) may then be restated as

(b′) *ch*(H)=1 entails H, and

(c′) *ch*(H)=0 entails ~H,

but not conversely. And here lies the rub. For while (a′) is no problem, there are obvious objections to both (b′) and (c′). Yet unless (b′) and (c′) – which, since they stand or fall together, I shall call collectively *the necessity condition* on chance – are satisfied, chances cannot embody factual possibilities. So as I think they do, I must meet these objections, which I shall do in §§5–6. But first I must clear the decks by distinguishing some other contingent and quantitative applications of ‘possibility’ that the probability calculus has also been used to measure.

2 Subjective and epistemic possibilities

Suppose I think that Pink Gin can win the Derby. What does my belief in this possibility amount to? Perhaps only to my being unsure that Pink Gin will not win. That, for Levi ([2] ch. 1), makes Pink Gin’s winning what Levi calls a ‘serious possibility’ for me, although I would rather call a possibility entailed merely by my believing it ‘subjective’. We can say therefore that, for any H and thinker X, H is *subjectively possible for X* iff X is unsure that ~H, i.e. iff X has a non-zero degree of belief in H.

Let us assume then, for the sake of argument, that belief can come by degrees with a probability measure [3], degrees which are therefore mostly called ‘subjective probabilities’ but which I, like Lewis [4], prefer to call *credences*. We may then take my credence in H – my *cr*(H) – to measure how possible I think H is. But why should we? One reason is given by Hart [5], who construes ‘probability as measuring uncertainty as to which is the actual world among the vast range of possible worlds’ (p. 287), i.e. worlds of which everything (K) I am sure of is true. And when there are only finitely many such K-worlds, Hart equates H’s subjective possibility for me with the fraction of these worlds that are also H-worlds, thus tacitly assuming, like Laplace ([6] ch. 1), that we take all such worlds to be equally possible.

But we need not buy Hart’s possible worlds and Laplacean metric to buy this modal reading of credences, for which we can give other reasons. One is that it explains the links with action that decision theorists like Jeffrey [7] use to define credences. Betting measures of credence, for example, assume that the lower my *cr*(H) the longer odds I will require for a bet on H, which makes good sense if my *cr*(H) measures how possible I think it is that I will win the bet. So if in particular I think I could not win it, because I think H is impossible, I should decline to bet on H at any odds, which is just what the theory says a zero *cr*(H) will make me do. And if I am a Bayesian, then no evidence E that does not entail H will alter my zero credence in H: for any experience that raises my *cr*(E) to 1 will then make my new *cr*(H) equal my old *cr*(H&E)/*cr*(E); and this, if my old *cr*(H) is zero, will also be zero. That commitment to being unmoved by all but logically compelling evidence for H makes little sense unless I think H is impossible.[8]

Finally, credences meet subjective versions of our conditions (a)–(c). That consistent credences meet (a) is obvious, since their probability measure makes them all lie between 0 and 1 and be such that, for all H, *cr*(H)+*cr*(~H)=1. So provided I have *some* *cr*(H), i.e. have thought of H at all, then if my *cr*(H) is greater than zero, consistency requires my *cr*(~H) to be less than 1, implying that I am not sure that ~H and so do not think ~H is necessary.

To see how credences meet (b) and (c) we must first ask how high my *cr*(H) has to be before I believe H. It may not need to be 1, since large prizes can make people bet against things they seem to believe, e.g. that they will not win a lottery. How much less than 1 my *cr*(H) can be before I stop believing H is then a tricky question, whose answer may well be vague and vary with H. However, all that matters here is that if my *cr*(H) *is* 1, and I am rational, i.e. have consistent credences, then I do believe H. So consistent credences do meet condition (b) in the subjective sense that any H that I think is necessary I must also think is actual, though not of course *vice versa*. Similarly, as I cannot consistently believe H unless my *cr*(H)>0, any H that I think is actual I must also think is possible – but again not *vice versa* – thus meeting a subjective version of condition (c).

As for subjective possibilities, so for epistemic ones. Calling H *epistemically possible* we may take to mean that the actual or available evidence E does not rule it out. What evidence is, and how it can do more than rule things out, are questions that fortunately we need not tackle here. All we need assume here, and again only for the sake of argument, is that E *can* do more than rule things out, and in particular that it can warrant non-zero credences in H which we may then use to measure how strongly E supports H. This gives us a probability measure of epistemic possibility, and with it a modal reading of the *epistemic probability* that E gives H, which I shall write '*ep*(H,E)'. For present purposes we need no other contentious assumptions about evidence, e.g. that all truths of the form '*ep*(H,E)=*p*' are necessary, or are equivalent to the Bayesian's '*cr*(H&E)/*cr*(E)=*p*'. All we need to mean by '*ep*(H,E)=*p*' is that it is true iff *cr*(H)=*p* is warranted in some way by the evidence E.

Does epistemic probability, so minimally understood, meet epistemic versions of (a)–(c)? It does of course meet (a), since *ep*(H,E)>0 iff *ep*(~H,E)<1. This means that if *ep*(H,E) exists at all, E will leave H epistemically possible iff it fails to make ~H epistemically necessary. Epistemic probability will also meet (c) if we may call H 'epistemically actual' iff belief in H, i.e. a high *cr*(H), is warranted. For then, as a high credence in H cannot possibly be warranted by evidence E which rules H out, i.e. which warrants *cr*(H)=0, any H that is epistemically actual must also be epistemically possible. Similarly for (b), since evidence E such that *ep*(H,E)=1 will automatically warrant full belief in H by warranting *cr*(H)=1, thus making any H that is epistemically necessary epistemically actual.

3 Factual possibilities

In these ways, with a little charity and ingenuity, we may condone the common practice of calling whatever it is that credences and epistemic probabilities measure 'possibilities'. Yet of course neither of them either is or entails the real McCoy. For, with a few Cartesian exceptions (like 'I exist' and 'I think') no H is made factually possible either by my thinking that it is or by my evidence E failing to rule it out. We must not mistake merely subjective or epistemic 'possibilities' for the real ones that I take chances to embody.

But do any such real factual possibilities exist? We certainly talk as if they do. Suppose I say that a coin toss could land heads (H). This alleged possibility is not subjective: the toss could land heads even if no one has a non-zero *cr*(H). Nor is it epistemic, since it is consistent with evidence – e.g. that the toss actually lands tails – which warrants *cr*(H)=0. If this possibility exists, it is clearly a fact, not about what anyone believes, or about what is evidence for what, but about the world. The fact may of course be denied, e.g. by determinists. But if it *is* a fact, then it can explain other facts, like the fact that some such tosses *do* land heads, which no subjective or epistemic possibility could explain but which we do take this possibility to explain. Why then should we deny that it is, as it seems to be, a real factual possibility?

The most common objection to such possibilities is ontological. What *are* they: what on earth can make it factually possible to some degree that a coin toss will land heads? In the actual world the toss may or may not land heads (H): if it does, that makes 'H' true; and if it does not, that makes '~H' true. What else is there, besides the toss landing (or not landing) heads, that could also make H more or less possible as a matter of contingent fact?

One obvious answer to this rhetorical question is that H is *physically* possible, meaning that it is consistent with the laws of physics or of nature generally. This however will not do. For first, as consistency does not come by degrees, it does not explain how H can be more or less possible and so have a greater or lesser chance. And second, no statements, not even law statements, can entail that every H which is consistent with them is possible, i.e. that nothing else can rule it out. (Take the law, $t=2\pi\sqrt{(l/g)}$, relating a simple pendulum's length l to its period of oscillation t under gravitational acceleration g. The existence of a negative square root of l does not make this law entail that a pendulum could have a negative period of oscillation!) 'Physically possible', meaning 'consistent with all laws', therefore no more entails even the negative possibility mentioned in §1, let alone the positive possibility embodied in non-zero chances, than 'physically necessary', meaning 'entailed by laws', entails necessity (since the laws themselves may not be necessary).

Another obvious answer to the question, anticipated in §2, invokes possible worlds, e.g. by equating our coin toss's *ch*(H) with the fraction of possible worlds, that are just like ours up to the toss, in which the coin does land heads. But this will not do either, not only because the answer's metric, and its ontology of possible worlds, face other objections, but because it concedes too much to the question. One need not be an actualist, i.e. believe that only our world exists, to reject the question's tacit assumption that chances are not features of single worlds. For why may not something in our world give a coin toss a non-zero chance of landing heads? As I see no reason why not, I take the question of what does so to be not rhetorical but serious, a question that needs a true answer if this factual possibility is to exist.

About contingent statements of chance I am therefore not just a 'realist', meaning that I think these statements 'are true or false in virtue of a reality existing independently of us' ([9] p. 146). I also require the relevant realities – the *truthmakers* of these statements – to be features of the actual and not just of possible worlds. The trivial equivalence between 'is possible' and 'exists in a possible world' may enable possible-world truth conditions to tell us what contingent chance statements mean, but they do not tell us what, in our world, makes statements with those meanings true.

4 Truthmakers for chance statements

Truthmakers are admittedly contentious entities (see e.g. [10] ch. 8 and [11] part III), about which I can say little here, except that I think only contingent truths need them. I can however say why true chance statements need truthmakers if any contingent truths do. One reason is the way chances are invoked to explain facts about frequencies: as when the chance $ch(\mathrm{H})=p$ of heads on a coin toss explains the fact (when it is a fact) that the frequency $f(\mathrm{H})$ of heads on many such tosses is close to p, because (given the laws of large numbers) the chance of this fact is close to 1. Obviously no credence or epistemic probability can explain why $f(\mathrm{H})\approx p$, since no such fact can be explained by how strongly anyone does or should believe in it. But a $ch(\mathrm{H})=p$ which embodies a factual possibility can explain it, since the more possible it is for each coin toss to land heads, the less possible it is for most of many such tosses not to do so.

But then to explain why $f(\mathrm{H})\approx p$ is a fact, $ch(\mathrm{H})=p$ must also be a fact. This is not just because the explanation is causal, although I think it is ([12] ch. 4.4), but also because we need such facts to specify the kind of tosses whose results define $f(\mathrm{H})$, namely tosses of which it is a fact that $ch(\mathrm{H})=p$.

An old if unvenerable retort to this is that chances do not *explain* frequencies, since they *are* frequencies: a toss's $ch(\mathrm{H})$ is the frequency $f(\mathrm{H})$ on many such tosses, where 'such' means sharing some other features but not a mythical $ch(\mathrm{H})$. But most philosophers of chance (e.g. [13] ch. 4.2) now agree that this retort will not do, as we can see by asking whether it identifies $ch(\mathrm{H})$ with an actual or a hypothetical $f(\mathrm{H})$. If actual, the number of actual tosses will restrict $f(\mathrm{H})$'s possible values, and hence those of $ch(\mathrm{H})$, which is absurd. (If there is only one toss, $f(\mathrm{H})$ can only be 0 or 1, which can hardly entail that the toss could have no other chance of landing heads.) While if $f(\mathrm{H})$ is hypothetical, we must ask what gives $f(\mathrm{H})$ its value. For now the tosses whose results define $f(\mathrm{H})$ need only be possible, and as *any* sequence of heads and tails is *possible*, so any $f(\mathrm{H})$, from 0 to 1 inclusive, is possible in any sequence of tosses, however long. So what in the world picks out the $f(\mathrm{H})$ that is to be the actual $ch(\mathrm{H})$, the chance of heads on the actual toss? The only possible answer is a quantitative property of the toss, with a value p given by the fact that, in an endless sequence of tosses with that very property, $f(\mathrm{H})$ would tend to a limit $f_\infty(\mathrm{H})=p$. But this property of a coin toss is just what I mean by $ch(\mathrm{H})$, the toss's chance of landing heads.

So while $ch(\mathrm{H})=p$ does indeed entail $f_\infty(\mathrm{H})=p$ for all p, this does not make the former reduce to the latter. Indeed it cannot. For as there need be no *actual* infinite sequence for $f_\infty(\mathrm{H})=p$ to be a property of, in no truthmaking sense of 'fact' need there be any such fact as $f_\infty(\mathrm{H})=p$. It must therefore be the fact that an actual toss has the property $ch(\mathrm{H})=p$ which makes '$f_\infty(\mathrm{H})=p$' true, not the other way round.

This shows incidentally how misleading the surface grammar of chance statements is. For what we *say* has the chance $ch(\mathrm{H})$ is the fact or event H, e.g. a coin toss landing heads. Yet as $ch(\mathrm{H})$ can exist even when H does not – a toss can have a chance of landing heads even if it actually lands tails – $ch(\mathrm{H})$ cannot be a property of H. It should therefore be obvious that $ch(\mathrm{H})$ is always a property of something else: in this case, as we have just seen, of the coin toss itself, since that is what must exist for this $ch(\mathrm{H})$ to exist.

In this respect, if no other, $ch(\mathrm{H})$ is like X's credence $cr(\mathrm{H})$, which is also not a property of H. That is why, as everyone knows, H can have many subjective probabilities, since all this means is that different people believe H to different degrees. Yet it is less widely realised that H can also have many chances, and for a similar reason: namely, that each of its chances is a property of a different fact or event. This is why a coin with a 50% chance of landing heads when tossed can also have a 90% chance of landing heads as it falls, heads up, just before it lands. There is no contradiction in this, since the first $ch(\mathrm{H})$ is a property of how the coin is tossed and the second a property of how it falls: these two different chances of a single toss landing heads are simply properties of different facts.

This underrated fact about chances also makes our coin toss's 50% chance of landing heads when tossed compatible with so-called hidden variables, i.e. further facts about the toss that either make or stop it landing heads by giving that outcome a chance of 1 or 0. There is no contradiction here either, even on the view that non-zero chances are real possibilities. For a toss's landing heads can easily be left possible to some extent by one fact about the toss, to a different extent by another, and made either necessary or impossible by a third. All there cannot be, since H cannot both exist and not exist, are two facts such that one makes H necessary and the other makes it impossible. Apart from that, the existence of many different chances of H is no more problematic than that of many people with different credences in H. (For a fuller defence of this unorthodox view, see my [12] ch. 5.2-3.)

5 Principled objections to the necessity condition

That H can have many chances, each a property of a different fact, is thus, if surprising, not at all paradoxical. Nor, once we realise that any factual possibility of H must also be a property of something other than H – since it too may exist even if H does not – is the idea of H having many such possibilities. The plurality of H's chances is therefore no reason to deny that each of them embodies a distinct factual possibility of H. What then are the objections to that idea, and in particular to the necessity condition which it entails, namely that, for all H, (b′) *ch*(H)=1 entails H and (c′) *ch*(H)=0 entails ~H?

One objection stems from misreading the so-called sample points in sample spaces to which chances are ascribed. If for example I think a tossed coin could conceivably land on edge, I may take the sample space for a toss to contain three points, heads, tails, and on edge (E), even though I think *ch*(E)=0. Does this not refute (c′) by showing that a possible fact can have a zero chance? Not at all, for the kind of possibility implied by E's being a sample point is not the kind that chances embody. All that my making E a sample point implies is that I think it *conceivable* that *ch*(E)>0, as indeed it is, since the factual impossibility of E which is embodied in the fact that *ch*(E)=0 is only contingent and thus not provable *a priori*.

Another bad reason for denying the necessity condition is its inconsistency with a frequency theory of chance mentioned in §4. This is the theory which identifies a coin toss's chance of landing heads, *ch*(H), with the limit f_∞(H) to which the frequency f_n(H) of heads in a hypothetical sequence of *n* such tosses would tend as $n \rightarrow \infty$. It therefore identifies *ch*(H)=1 with f_∞(H)=1 and *ch*(H)=0 with f_∞(H)=0. But as f_∞(H) is only f_n(H)'s *limit*, it could be 1 even if many tosses in an endless sequence did *not* land heads, and 0 even if many did. So f_∞(H)=1 does not entail H and f_∞(H)=0 does not entail ~H.

Thus if chances were limiting frequencies they could not meet the necessity condition. But chances, as we saw in §4, are *not* limiting frequencies: they merely entail them. And to that entailment we can simply add the necessity condition, thus making *ch*(H)=1 entail H as well as f_∞(H)=1, and *ch*(H)=0 entail ~H as well as f_∞(H)=0. The fact that limiting frequencies do not meet the necessity condition is no reason to deny that chances do.

6 Counter-examples to the necessity condition

These two objections of principle to the necessity condition are thus easily met. What really threaten the condition are not conflicting principles but several kinds of apparently intractable counter-examples. As these need different treatments, I shall take them in turn.

1 Spinning pointers

Suppose that, as it is spun, a spinning pointer has an equal chance of stopping in any equal-angled sector of the circle it marks out. This makes its chance of stopping within any angle α proportional to α, and its chance of stopping at any one point zero. Yet the pointer must stop somewhere. How then can its zero chance of stopping at a point entail that it does not do so?

Lewis, who accepts the necessity condition, lets his pointers stop at points by giving them infinitesimal chances of doing so ([4] p. 89). But it is within infinitesimal angles, not at points, that pointers have infinitesimal chances of stopping. So this answer will only work if chances can be infinitesimal but angles cannot, which I see no independent reason to believe. This is why in my ([12] ch. 3.1) I preferred to make the more credible assumption that any pointer, however thin, must have *some* width. If we measure this by the small (and perhaps even infinitesimal) angle $\delta°$ its end subtends at its pivot, the nearest it can get to stopping at a point is by stopping at any segment of angle δ° which includes that point, and its chance of doing that is not zero but δ/360.

But why can the pointer's centre line or edges not stop at a point? Because, I say, since no physical boundary has *no* thickness, even edges (and hence centre lines) must have some width. But what then of the edges' edges, the edges of those edges, and so *ad infinitum* (like the lesser fleas that all fleas have 'upon their backs to bite 'em')? Here I admit I may well need infinitesimals, which also come in endless sequences of ever-smaller but still non-zero sizes ([14] Appendix 4). This lets me credit all edges, edges of edges, etc. with widths that are at least infinitesimal, thereby giving them all non-zero chances of stopping at a point by stopping at some short segment including that point.

2 Continuous quantities

This solution to the spinning pointer example also works for other quantities whose values can have chances. Take the temperature θ at any time *t* of a gas sample *g*, and suppose, as statistical mechanics implies, that at a slightly earlier time t-ε there is a chance distribution over *g*'s possible values of θ at *t*. Then, as with our pointer, *g* has at t-ε a zero chance of having at *t* any one point value of θ. Yet *g* must have *some* temperature at *t*. So to be able to have chances which meet the necessity condition, temperatures and other continuously variable quantities must also be confined to interval values, as indeed modern physics says they are. To be at 25° is therefore not to lack all other temperatures, even infinitesimally close ones, but to have a temperature that is some interval, however small, which includes 25.000…°.

3 Decaying atoms

Another quantity which needs this treatment is the time a radioactive atom takes to decay. Its chance $ch(\mathrm{D}_t)$ at any time of decaying within the next time interval *t* is $1\text{-}e^{-\lambda t}$, where λ (>0) is its so-called decay constant, a chance that is zero if *t* is zero. If time were discrete this would not matter, since there would be a least time interval, and the atom's having a non-zero λ at any instant would give it a non-zero chance of decaying at the next instant. But if time is dense, as no doubt it is, there is never a next instant or a least time interval, and that makes our atom's $ch(\mathrm{D}_0)=0$ like our pointer's zero chance of stopping at a point. So here too I say it cannot be done. All physical processes, and all their temporal boundaries, take some time, however short. No atom can decay, start to decay, start to start to decay, etc., at an instant, but only in some short, and perhaps infinitesimal, interval of time.

Our atom also poses a longer-term problem, since as $t\rightarrow\infty$, $1\text{-}e^{-\lambda t}\rightarrow 1$, so that $e^{-\lambda t}$ and hence $ch(\sim\mathrm{D}_t)\rightarrow 0$. That seems to give a radioactive atom a zero chance of lasting for ever, and hence to imply that it cannot do so. This however is not a bullet I need to bite, for since $ch(\sim\mathrm{D}_t)>0$ for all finite *t*, the atom can last – and can have lasted – longer than any finite time, however long: there is no upper limit to its possible present or future age. What more can it mean to be able to last for ever?

4 *Multiple coin tosses*

If that rhetorical question can disarm this apparently zero chance of an apparent possibility, there is an even better way of disarming some others. Take the chance $ch(H_n)=p^n$ of n coin tosses all landing heads (H_n) when each $ch(H)=p$ ($0<p<1$), which also $\to 0$ as $n\to\infty$. Yet as each toss in this possibly infinite sequence can land heads, so can they all: H_∞ is possible. How can we square this possibility with the necessity condition?

Perhaps, as in §6.3, we could settle for there being no finite limit to how many tosses can all land heads. But here we can do better than that. The reason is that whereas an unstable atom's zero chance $ch(\sim D_\infty)$ of surviving for ever may follow from a single fact (that it has a non-zero decay constant λ), the zero $ch(H_\infty)$ may follow only from the infinity of facts that $ch(H)=p$ on each toss.

To show why this difference matters I need another assumption about the truthmakers mentioned in §4, an assumption that makes 'facts' a bad name for them. For while we standardly govern our use of the term 'fact' by the principle that, for any H, H is a fact iff 'H' is true, the only truths that need truthmakers are not only contingent but *atomic*. For if truthmakers P and Q make 'P' and 'Q' true, they will also make true 'P&Q' and, for any R, 'PvR' and 'QvR'. Similarly, what will make 'P' false and thus '~P' true is not that a corresponding truthmaker (~P) exists but that P does not. So we need no conjunctive, disjunctive or negative truthmakers to make the corresponding molecular propositions true, which is why I say there are none. That is why, to avoid confusion with the more liberal, standard and still useful use of 'fact', I now call truthmakers not 'facts' but 'facta' ([12] ch. 13.4).

In particular, therefore, if n tosses all land heads, the conjunctive proposition 'H_n' is made true not by a conjunctive factum but by the n facta that make 'H' true for each toss. Similarly, perhaps the contingent proposition that H_∞ has zero probability can be made true by the facta that give each toss a $ch(H)=p$, where $0<p<1$. But then, as none of those facta entails ~H, nothing need entail $\sim H_\infty$. If so, and H_∞'s zero probability is not itself a factum, but only a consequence of many other facta, it need not meet the necessity condition: it can entail $f_\infty(H_\infty)=0$ without entailing $\sim H_\infty$, a feature I shall mark by not calling it a chance and writing it not '$ch(H_\infty)=0$' but '$pr(H_\infty)=0$'. In short, perhaps H_∞ is negatively possible in the sense of §1, i.e. is possible not because it has a positive chance but because it is a logical possibility that has *no* chance, zero or otherwise.

The objection to this idea is that the fact that $ch(H)=p$ on each of n tosses does *not* suffice to entail that $pr(H_n)=p^n$. The tosses must also be *independent*, meaning that the value of $ch(H)$ on one toss must not depend on how any of the other tosses lands. Moreover, the objection runs, it takes causal facta (like the coin's being rigid) to make tosses independent in this sense, just as it takes causal facta (like the coin's being plastic enough for landing heads to bend it) to make them dependent. And the facta which do this are just like those that make an atom's chance $ch(D_1)$ of decaying on one day depend on whether it actually decays the day before, or a coin's 90% $ch(H)$ as it falls depend on how it actually moved just after it was tossed. Yet if these causal dependencies stopped an atom's $pr(D_2)$ of decaying in the next two days, or a coin toss's initial 50% $pr(H)$, being a chance, there would *be* no chances. So if those probabilities *are* chances, must not a coin's zero $pr(H_\infty)$ also be a chance?

No, for there are two relevant differences in this case. (1) The *n* facta that make $ch(H)=p$ on each of *n* tosses of a coin really are all the facta needed to make those tosses independent. The coin's rigidity could indeed make them independent, by making any toss's $ch(H)$ depend causally only on the last toss's $ch(H)$ and not on how many earlier tosses had landed heads. But this causal link is not necessary, as we can see by thinking of *n* independent tosses, of *n* different coins, that are simultaneous in some reference frame. Then as there arguably cannot be, and certainly need not be, any causation across spacelike intervals, these tosses cannot have to be made independent by causal links between them. All it takes to make them independent is that *no* causal links make them anything else. In short, it is only dependence which needs more facta than those that fix the value of $ch(H)$ on each toss: all independence needs is that no such other facta exist.

(2) Single coin tosses get their $ch(H)=p$ from facta about how coins are tossed, just as atoms get their $ch(\sim D_t)$ for all *t* from facta about their nuclear structure and the laws of radioactivity. But as we rarely set up devices to *go on* tossing coins, rarely for $n>1$ do any facta exist to give coins *any* present chance, zero or otherwise, even of being tossed *n* times, let alone of landing heads *n* times. But if no $ch(H_n)$ – and *a fortiori* no *zero* $ch(H_n)$ – exists for $n>1$, then for all such *n*, since H_n is logically possible, it will also be at least negatively possible in the sense of §1. The probability $pr(H_n)=p^n$ in this case is only hypothetical, the hypothesis being simply that $ch(H)$ *would* be *p* ($0<p<1$) on each such toss if it occurred. For given (1), and the fact that each toss need not but could land heads, then even if *n* could be infinite and make p^n zero, we need hypothesise no factum that would stop all those tosses landing heads.

5 *Evolving fields*

Finally, consider how the spatial profile of a continuous quantity like temperature may evolve over time. Even if laws of nature make such quantities vary continuously across space and time, they need still not evolve deterministically: their temporal and/or spatial gradients (or their gradients' gradients, or …) at any spatial point *s* at any time *t* may still be like the results of coin tosses. So suppose they are, and let ϕ be such a variable, with a chance distribution at a slightly earlier time t-ε over its possible values at *s* at *t*.

Then the spatial profile of ϕ at *t* is a continuous analogue of a discrete spatial profile of heads and tails following a spatial array of *n* simultaneous coin tosses. H_n is the analogue of a flat ϕ-profile, as other profiles of heads and tails are of varying ϕ-profiles. Yet with only these two possible outcomes of a toss, but infinitely many values of ϕ (and spatial points to have them), possible ϕ-profiles far outnumber the possible profiles of heads and tails. So whatever the actual ϕ-profile at *t*, its probability at any earlier time will almost certainly be zero. Yet some such profile there must be. How can this be?

To see how, consider s_ε, the region of space in *s*'s backward light cone at *t*-ε, and suppose that everything at *t*-ε which affects anything at *s* is in s_ε. This will then include all the facta that fix the chances at *t*-ε of the possible values of ϕ at s. So if, for example, ϕ is a temperature gradient, and it depends only on earlier temperatures, the facta that fix these chances will be those that fix the temperature profile across s_ε plus the supposed probabilistic laws of thermal evolution.

These chances at *t*-ε of the possible values of ϕ at *s* are just like a single coin toss's chances of landing heads and tails, except that *s* has not two but infinitely many possible values of ϕ. Thus just as the spinning pointer of §6.1 has a zero chance of stopping at any one point, so there is a zero chance at *t*-ε of *s* having any one point value of ϕ. Here therefore, as in §6.2, I conclude that *s* can have only interval values of ϕ.

And as for *s*, so for all other spatial points at *t*, the conjunction of whose values of ϕ is a spatial ϕ-profile at *t*, just as the conjoined results of infinitely many simultaneous coin tosses is a spatial profile of heads and tails. In each case, as the number of conjuncts is infinite, the probability of any one such profile may be zero. But since the profile is logically possible, this zero probability need not make it impossible. For as in the coin tossing case of §6.4, the probability at *t*-ε of any ϕ-profile at *t* need not be a chance. The only facta needed to make true the proposition that this probability is zero are those that give all points at *t* their non-zero chances at *t*-ε of having various interval values of ϕ. And as none of those facta stops *s* having any such value, nothing need rule out any ϕ-profile that is a conjunction of them.

7 Conclusion

These are my answers to the objections I know of to the necessity condition that chances must meet if we are to interpret them as factual possibilities. But are not these complex answers to so strikingly wide a range of counter-examples suspiciously *ad hoc*? I say not, since what is really striking is that all the examples can be disposed of with just two theses, both of which we have other reasons to accept:

(A) *Particulars cannot have point values of continuous quantities over which they have chance distributions*. This thesis is supported not only by modern physics but also by the lack of any reason to credit particulars with point values of continuous quantities in the first place. We need no theory of chance to tell us that particular things and events never have positions, temperatures, masses, volumes, etc., which it would take more than a finite number of significant figures to distinguish from positions, temperatures, etc. that they do *not* have.

(B) *There are no molecular truthmakers*, and especially no conjunctive or negative ones. This thesis is indeed contentious, and may need more argument than I have given in this paper. But as no such argument depends on any view of chance, at least none begs any question here.

This being so, not only is the defence that (A) and (B) provide for the necessity condition not *ad hoc*, but the reasons I have given for the view of chance which entails that condition are also reasons for accepting (A) and (B). To those reasons I myself would add the way this view enables chances to explain the core consequences of both deterministic and indeterministic causation: namely, that causes explain their effects, are evidence for them, and provide means of bringing them about. For as I show in my ([12] chs 6–7), none of these consequences will follow unless (i) causes raise the chances of their effects, ideally to 1, and (ii) thereby eliminate or at least reduce a factual possibility that their effects will not occur. All this, and the natural account which the necessity condition yields of how sufficient causes of any effect E make it factually necessary, by making *ch*(E)=1, give us an even stronger reason to accept the condition.

However, while chance's role in causation remains controversial, I fear it may convince fewer people than it should of the necessity condition on chances. So, to supplement it, let me end by putting the general case for my modal view of chance in three rhetorical questions:

(1) What, if not chances, embody the quantitative factual possibilities the world clearly contains?
(2) What can chances be – since they clearly cannot be credences or frequencies – if not the embodiments of such possibilities?
(3) Why do these possibilities accompany chances if chances do not embody them?

It is because I see no good answers to these questions that I see no real alternative to this view of chance, regardless of its role in causation, nor hence to the necessity condition that the view entails.[*]

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[*] This paper is descended from many very varied forbears discussed and often demolished in the last five years in seminars at the Universities of Belgrade, Cambridge, Colorado, Columbia, Keele, London (UCL and KCL), Lund, Queensland, Sydney and Western Australia, and the 1996 Biennial Meeting of the Philosophy of Science Association. I am indebted to those who put me right on those occasions, to private conversations and correspondence with Alexander Bird, Nick Denyer, Matthias Hild and Arnold Koslow, and to the reports of this journal's anonymous referees. But by far my greatest debt is to Kieran Setiya: this paper owes so much to his comments on earlier drafts that whatever is right in it is as much his work as mine, and only at his insistence is he not a named co-author.

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