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# Chances and Conditionals

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In a forthcoming book, *Most Counterfactuals Are False*, Alan Hájek infers the truth of its title from the ubiquity of chance. In this paper I argue that the inference is invalid: chances do not falsify counterfactuals.

## 1 Chances

By 'chances' I mean the empirical probabilities postulated by theories in physics, genetics, evolution, epidemiology, etc., to explain approximately stable frequencies, like the proportions of radium atoms decaying in a given time, of human births that are male, and so on. But whether chances, so understood, could falsify counterfactuals depends on what they are taken to be, which remains a contentious matter. Russell (1948 pt. V ch.VIII) takes them to include actual frequencies, like the proportion of male births in the UK in 2016, as well as the theoretical chances postulated to explain them. For von Mises (1957 pp.14–15) and others, chances are the *limits* to which actual frequencies, e.g. of male births in ever larger populations, would tend if those populations increased indefinitely.

It is obvious that no such frequency theory can make chances falsify singular conditionals. The actual or limiting value of the frequency *f*(H) of heads in many coin tosses, for example, may provide inductive *evidence* for or against the singular conditional

C 'This coin will land heads if tossed' or, for short, 'If T then H',

or its conditional negation –

¬C 'This coin will not land heads if tossed' or, for short, 'If T then ¬H',

but it can hardly entail their falsity.

The only theories that might make chances falsify C or ¬C are the 'single-case' theories (Eagle 2018 §1.10) which take a coin toss's chance *p* of landing heads to be a property of *that very toss*: namely, the property such that a sequence of *f*(H)s in ever larger classes of tosses *with that property* would have a limiting value *p*. This property of a single coin toss – that it has a chance *p* of landing heads[1] – might well conflict with a conditional like C or ¬C which says that a coin definitely will, or definitely will not, land heads if tossed. That is why in what follows I shall take for granted a single-case theory of what chances are – a theory I accept anyway – after making two points about it.

First, the more independent coin tosses with the same chance *p* of landing heads, the less chance the *f*(H) in those tosses has of differing from *p* by any given amount, however small (von Mises 1957 pp. 126–7). While

[1] To avoid irrelevant complications, I assume throughout that *p* is less than 1 and greater than 0.

this will not tell us how close to *f*(H) we can infer *p* is – it takes contentious theories of statistical inference to tell us that – it does indicate why, the more tosses we observe, the better *f*(H) is likely to be as an estimate of *p*.

This in turn prompts the other question I need to raise: how can we use a frequency of heads in many coin tosses to measure *p* if, before we can do so, we must know that each toss has the same chance – as yet unknown – of landing heads? That is a fair question, but not peculiar to measurements of chance. How, for example, can a thermometer tell us an air temperature *Θ* if, before it can tell us what *Θ* is, we must know that it is *at Θ*? The answer is that, if we know what would make the thermometer hotter or colder than the air it's in, we can ensure that it isn't: e.g. by sheltering it from sunlight that would make it hotter, or from cold rain that would make it colder, before reading it.

Similarly with chances, which is why theories that postulate chances postulate laws which make those chances functions of other properties. A radioactive atom's chance of decaying in any given time is a function of its nuclear structure; our chances of catching infections we are exposed to are functions of our genetic and other properties; and so on. And similarly for the chance *p* which, for the sake of a simple exemplar, I am supposing that a single coin toss has of landing heads. That chance *p* will, we can assume, be determined by a limited number of the coin toss's other properties – properties of which, if we used a frequency *f*(H) of heads to measure *p*, we would tacitly assume *p* to be a sufficiently constant function.

## 2 Conditionals

The question then is whether single-case chances falsify simple conditionals like C, 'This coin will land heads if it's tossed': namely, conditionals with singular antecedents and consequents that are unconditional, contingent and independent. Necessary conditionals like 'If the coin landed heads it landed', and complex ones like 'If it were to land on edge if it was tossed, I'd eat my hat if I had one', we can, fortunately, ignore.

However, whether chances falsify even conditionals as simple as C depends as much on the right theory of them as on the right theory of chance. So in order to answer the question I must first outline and defend my own theory of these conditionals (Mellor 1993), which develops the descriptive core of Stalnaker's (1984) thesis that they express inferential dispositions. On this theory, applied to the less blatantly chancy example,

'If I take exercise I shall get fit' or, for short, 'If E then F',

to accept 'If E then F' is to be disposed to infer 'F' from 'E', i.e. to be in a mental state which will make my coming to believe 'E', that I am taking exercise, cause me to believe 'F', that I will get fit.

And if I will in fact get fit if I take exercise, then this disposition will not make a true belief in 'E' cause a false belief in 'F'. In other words, 'If E then F' will, in that case, be truth-*preserving* – or, for short, *safe*. Then I say that what makes this disposition, and the conditional that expresses it, objectively right, are that they are, in this sense, safe.

Whether being safe is enough to make a simple conditional *true* is a question I shall not try to answer here, since it remains too contentious whether conditionals even *have* truth values, let alone what fixes them. So in what follows I shall evade the question by asking not whether chances make simple conditionals *false* but whether they make them *unsafe*: since that, on my theory, is what matters.

I start with two obvious points about the safety of simple factual conditionals, i.e. ones with true antecedents: for all 'P' and 'Q', then

(1) if 'P' is true and 'Q' is false, 'If P then Q' is unsafe, i.e. not truth-preserving;

and the analogue for safety of Lewis' (1973 p. 14) 'centering' principle for truth, namely that

(2) if 'P' and 'Q' are both true, 'If P then Q' is safe.

What about counterfactuals? Suppose, in the coin-tossing example C, 'If T then H', that the coin is not being tossed, i.e. that 'T' is false. In that case, whether 'If T then H' is safe depends on whether 'H' *would* be true if 'T' was: i.e., on whether the non-actual world that a merely possible coin toss would take us to is what I shall call an 'H-world', i.e. one where the coin lands heads, or a ¬H-world, where it does not land heads. Now even if there is no saying which one of myriad possible T-worlds (worlds where 'T' is true) a non-actual coin toss would take us to, it could not take us to *more* than one. And in whatever world it would take us to, the coin will either land heads or it will not: so if 'H' is not true there, '¬H' will be. In short, even when C, 'If T then H', and ¬C, 'If T then ¬H', are counterfactual, one but not both of them will be safe.

This of course conflicts with what most theorists say about the truth of C and ¬C. Some deny either that C and ¬C *ever* have truth values (Edgington 1986) or that they have them when they are counterfactual (Edgington 2005). Others (Jackson 1990) read C and ¬C as *material* conditionals which, if counterfactual, will both be true. For Lewis (1973), by contrast, C and ¬C will be *false* if they are counterfactual if, as I am assuming, our coin has a chance $p$ of landing heads if tossed: since that, by making 'H' false in some but not all T-worlds that are most like our world, will, on Lewis's theory, make C and ¬C false.

I reject all these theories because, I shall now argue, they make no sense of the role of conditionals in subjective decision theory (Mellor 2005b): the theory of what *will* (or, on normative readings, *should*) make us do something as a means to an end: like my taking exercise in order to get fit. And while the theory applies even when I am uncertain if my action will succeed, all my argument needs is the simple case where I *am* certain my action will succeed.

## 3 Conditionals and Decisions

Suppose then that I am wondering whether to take exercise, E, in order to get fit, F, when I am quite sure that I *will* get fit, which I would like, if and only if I take exercise, which I dislike. In this situation, subjective decision theory says that whether I will (or should) take exercise depends on two factors. One is how much I value or disvalue four possible scenarios: E&F, I take exercise and get fit; E&¬F, I take exercise but do not get fit; ¬E&¬F, I do not take exercise and do not get fit; and ¬E&F, I do not take exercise but get fit anyway. And as I should much prefer the last of these, only if something rules it out will I (or should I) exercise to get fit.

What rules it out, of course, is my accepting the conditionals ‘If E then F’ and ‘If ¬E then ¬F’, which is the other factor my decision depends on. This by reducing my foreseen possible outcomes to two, E&F and ¬E&¬F, is what subjective decision theory say will (or should) make me take exercise, i.e. make ‘E’ true, if I value getting fit – F – more than I disvalue taking exercise – E.

So if we now ask what, in these circumstances, makes it *right* for me to take exercise, the answer is that ‘If E then F’ and ‘If ¬E then ¬F’ are the right conditionals for me to act on. And what makes them right, of course, is that they are *safe*, i.e. that I will get fit if I do take exercise, and won’t if I don’t, and their conditional negations, ‘If E then ¬F’ and ‘If ¬E then F’, are *unsafe*. Moreover, since these four conditionals’ ‘safety values’ (as I shall call them) are what, given my likes and dislikes, make taking exercise the right thing for me to do *whether I do it or not*, their safety values must be independent of whether or not I do take exercise, i.e. of whether I make ‘E’ true or false. And so they are on my theory, since ‘If E then F’, for example, will be safe if and only if ‘F’ is true *if* ‘E’ is true: whether ‘E’ *is* true is immaterial, and similarly for the other three conditionals.

This independence condition, however, is *not* met by the theories cited above that take *truth* to be what makes conditionals objectively right. For on the one hand, a material conditional that is *false* if it’s factual will be *true* if it’s counterfactual (since all material counterfactuals are true); while on the other hand, a Lewis conditional that is *true* if it’s factual will, if there are relevant chances, be *false* if it’s counterfactual. It is because these theories make the truth values of ‘If E then F’ and ‘If ¬E then ¬F’ depend on those of their antecedents, that they cannot make truth be what, given my likes and dislikes, makes it right for me to take exercise whether I do so or not. And nor of course can theories on which counterfactuals *lack* truth values. Only if safety is what makes conditionals right can the decisions they mandate inherit their rightness.

The main objection to this argument rests on the normative reading of subjective decision theory mentioned earlier, a reading which tells us to act on the conditionals we accept, since they are after all the only ones we *can* act on. This in my case tells me to take exercise if I accept ‘If E then F’ and ‘If ¬E then ¬F’ and not to take it if I do not accept them. That, it says, is the rational and thus the right thing to do, whether or not those conditionals are safe.

I disagree, for reasons given in my (2005b §5). There is more to being right than being subjectively rational, as the evident error of accepting and acting on the unsafe ‘If E then ¬F’ and ‘If ¬E then F’ shows. What makes it right to act on ‘If E then F’ and ‘If ¬E then ¬F’ is not that I *think* I will get fit if I take exercise and won’t if I don’t, but that I *will* get fit if I take exercise and won’t if I don’t, i.e. that ‘If E then F’ and ‘If ¬E then ¬F' are safe, and their conditional negations are unsafe. That is why *epistemically* rational agents will try, before acting, to make sure that the conditionals they act on are safe.

## 4 Chances and Counterfactuals

Given this safety theory of conditional rightness, and a single-case theory of chance, I can now turn at last to the *right* question about chance: does it make conditionals like C and ¬C unsafe? That it does not do so when they are factual is obvious, as the centering principle shows: if a coin *is* tossed and *does* land heads, so that 'T' and 'H' are both true, then 'If T then H' will be truth-preserving, whatever its chance of being so. The only question is whether chances make counterfactuals unsafe: does a coin's chance of landing heads if tossed make 'If T then H' unsafe when 'T' is false?

To answer that question I must distinguish what I shall call the '*counterfactual*' chance of landing heads that an untossed coin *would* have if it *was* tossed from the coin's *conditional* chance of doing so. The latter is the application to chance of a coin's *conditional probability* of landing heads if tossed, defined as its unconditional probability of being-tossed-and-landing-heads, T&H, divided by its unconditional probability of being tossed, T (Mellor 2005a ch. 1 §VII). Since conditional chances, so defined, are entailed by actual chances, in this case by the chances of T&H and of T, identifying them with counterfactual ones implies that they too are entailed by actual chances. This implies that those chances fix the coin's counterfactual chance of landing heads if tossed, regardless, for example, of *how* the coin is tossed, which is absurd.[2]

I am not sure why this implausible identification of counterfactual with conditional chances is so widely accepted. I suspect the reason is the widespread failure to realise how differently the probability calculus can apply to different kinds of probability: in this case, to chances on the one hand and, on the other, to *credences*, the probabilistic degrees of belief postulated by subjective decision theories to explain action under uncertainty. Suppose for example you see a coin being tossed (but not how it lands) and this convinces you that it has been tossed, i.e. raises to 1 your lower 'prior' credence in T. Then for Bayesians (Jeffrey 1983 ch. 11.1) this change in your credence in T should – and if you are rational will – turn your prior credence in H, that the coin landed heads, into a 'posterior' credence in H equal to your prior *conditional* credence in H.

Now whether or not you buy this normative application of conditional probability to credences – I do not, for reasons given in my (2005b) – it does at least make sense. Applied to chances it's nonsense. Whether a coin's counterfactual chance of landing heads if tossed can be identified with its conditional chance of doing so is a matter not of rationality but of fact. And, as I have noted, as a matter of fact it can't. The chance of landing

[2] It does not help to make conditional probability a primitive concept and define an unconditional probability as a probability conditional on a tautology, because actual chances are not conditional on anything. The only chances that could be conditional are counterfactual ones, i.e. what a chance would be if …. But identifying those with primitive conditional probabilities removes the whole point of the identification: namely, to enable counterfactual chances to be be entailed by actual ones, which they aren't.

heads that an untossed coin *would* have if it *was* tossed does not depend at all on its actual chances of being tossed and/or of landing heads: all it depends on is how the coin would be tossed if it was tossed. When

C*p* 'If the coin's tossed it'll have a chance *p* of landing heads'

is counterfactual, its safety values is as independent of actual chances as is that of

C 'If the coin's tossed it will land heads'.

## 5 Chance and Determinism

The fact that the safety values of the counterfactual C*p* and C are independent of actual chances does not of course show that they are independent of each other. The question still remains: does C*p*'s safety make C unsafe: does a coin's having a counterfactual chance of landing heads if tossed make 'If T then H' unsafe when 'T' is false? In particular, does it rule out a hidden variable, a property *D* that makes all and only coin tosses which have that property land heads?

The quickest way to see that it does not is to compare chances with deterministic dispositions, and the conditionals they make safe. To be soluble, for example, is to have a property which makes things dissolve when put in water – provided that putting them in water does not make them *insoluble*, i.e. that their solubility is not what Martin (1994) calls 'finkish'. And this proviso, that solubility is not finkish, shows that the conditional which is made safe by a substance *x*'s solubility *Sn* of *n* grams/litre is not the simple

'If not more than *n* grams of *x* is put into 1 litre of water it will dissolve'

but the more complex conditional

'If not more than *n* grams of *x* is put into 1 litre of water *and is still Sn*, it will dissolve',

a conditional that I follow Carnap (1936–7) in calling a 'reduction sentence'.

Similarly with velocity. A train *y* moving at *n* miles/hour may not have gone *n* miles an hour later, because it may be speeding up or slowing down. So the conditional which is made safe by its velocity *Vn* of *n* miles/hour is not 'If it's an hour later *y* will have gone *n* miles' but the reduction sentence

'If it's an hour later a*nd Vn has not changed*, *y* will have gone *n* miles'.

This is what makes velocity compatible with acceleration: *y* can both have a property *Vn* which, if it persists for an hour, will move *y* on *n* miles, *and* a property *A* which, if *it* persists for an hour, will move *y* on *more* than *n* miles.

And so it is with chances. What makes single-case chances compatible with determinism is the fact that a single coin toss can belong to different classes of tosses with different frequencies of heads: a class of tosses with a property *D* that makes them all land heads; and a class of tosses with a chance *p* of landing heads which contains some that do not land heads. This is why, if a coin that is not being tossed *was* tossed, that merely possible toss's chance *p* of landing heads does not stop it also having a property *D* that will make it land heads, thereby making the deterministic counterfactual

C  ‘If the coin is tossed it will land heads’,

as safe as the chance counterfactual,

C*p*  ‘If the coin is tossed it will have a chance *p* of landing heads’.

There is, however, an objection (Hawthorne 2005 p. 396) to accepting both C and C*p* that I should meet at this point. This is that, since *p* is less than 1, C*p* entails ‘If the coin was tossed it *might not* land heads’, which contradicts the explicitly counterfactual ‘If the coin was tossed it *would* land heads’. But this ‘might not’ counterfactual can rule out its ‘would’ counterpart without making it *unsafe*: Grice’s (1975 p. 161) conversational maxim ‘Make your contribution as informative as is required …’ will make it do that anyway. For telling someone who asks how a coin toss will land that it *might not* land heads implies that you do not know, and therefore cannot honestly say, that it *would* land heads. That is quite enough to make ‘might not’ and ‘would’ conditionals conversationally incompatible; but it does not begin to show that they cannot both be safe.

## 6 Indeterminism and Knowledge

But what if determinism is false and there are no hidden variables? What if no property *D* of a coin toss makes all and only tosses with that property land heads: can C and C*p* still both be safe if they are counterfactual? I say they can. I argued in §2 that if C, ‘If T then H’, is counterfactual, i.e. if ‘T’ is false, then in whatever T-world a non-actual coin toss would take us to, the coin will still either land heads or not, thereby making safe either the counterfactual C, ‘If T then H’, or its conditional negation ¬C, ‘If T then ¬H’.

And as for C and ¬C, so for the counterfactual C*p* and its conditional negation ¬C*p*, ‘If the coin is tossed it will *not* have a chance *p* of landing heads’. For in whatever T-world a non-actual coin toss would take us to, it will in that world either have, or lack, a chance *p* of landing heads. And if it does have that chance, that can no more stop it landing heads in that world than it can in ours. That is why the counterfactuals C and C*p* can be as safe together without hidden variables as with them.

When C and ¬C are counterfactual, indeterminism may not stop one of them being safe, but it may stop us knowing which one *is* safe: knowledge that a property *D*, which makes all and only *D*-tosses land heads, can give us if, for example, we know enough about our coin-tossing device to know whether, if it did toss a coin, that coin toss would be *D*. But if there is *no* such property *D*, can we ever know which way a future or possible coin toss will or would land?

I say we can, if the coin toss’s chance of landing heads is close enough to 1 or 0. To show this, I need what in my (2005a ch. 6.IV) I call the ‘chances-as-evidence’ or ‘C-E’ principle, which says in this case that

if all you know about how a coin toss will land is that it has a chance *p* of landing heads,
then your *credence* that it will land heads should also be *p*.

For then, if $p$ is close enough to 1, I think we can know in advance that a future coin toss will land heads, or that a possible one would land heads, even if no present or actual hidden variable makes it do so.

Thus if, to vary the example, all I know about a future toss of a double-headed coin is that its chance of landing heads will be 0.99 (it *might* land on edge …), then the C-E principle says that my credence in its landing heads should also be 0.99. And this is so close to 1 that a normative decision theory will tell me to bet on heads, unless of course a £1 bet against heads would net me at least £100 if I won. So if how the coin lands matters less to me than that, as it usually will, then I say a 0.99 credence in heads, warranted by a known 0.99 chance of heads, can amount to knowing that the coin will land heads, provided of course that it does so.

Now suppose I see a toss of an ordinary coin land heads. Suppose too that my eyes, and the lighting, are good enough to give me a 0.99 chance of seeing *truly* how the coin landed, and that my seeing it land heads gives me a 0.99 credence that it *did* land heads. This will also count as knowing how the coin landed if not too much turns on it. And if more does turn on it, I can always look again, or more closely, to raise my chance of seeing truly how it landed, and my consequent credence that it landed heads, to as high a level as it takes – a level which, however high, will always be less than 1, for anyone unwilling to risk losing everything if they are wrong in return for an infinitesimal gain if they are right.

This I believe is how our fallible senses give us perceptual knowledge: by giving us chances of true perceptions, and consequent credences in those perceptions, which though less than 1 are still high enough in any actual context to warrant acting on them. In short, and in conclusion, not only does the ubiquity of single-case chances not show that most counterfactuals are unsafe, it does not even stop us knowing which ones are safe when there are hidden variables, and often when there aren't.

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