

Properties of Chance

1 Kinds of Probability

1. I'm going to talk about *chances*, i.e. probabilities of the kind postulated by theories in physics, genetics, evolution, epidemiology and other empirical sciences. The *first* thing to say about them is that they're not the *only* kind of probability. Other things are *also* called 'probabilities', simply because they too use a probability *measure* of their quantitative subject matters. By this I mean they use a measure that makes all its values lie between 0 and 1, and the value of *anything* be 1 minus that of its negation, or non-existence, or non-occurrence, depending on what kind of thing it is – which of course the measure itself doesn't tell us.

2. I emphasise this because too many authors write about probability without saying which kind they mean, as if it didn't matter, which it usually does. It matters here, for a start, because chances are the only probabilities that might threaten *determinism*. That's the first thing I want to show, by showing how chance differs from the two other kinds of probability in **Note 1**.

3. First, there are the *inductive* probabilities postulated by some inductive *logics* (e.g. Carnap 1962). These use a probability measure of how strongly some empirical evidence E *confirms* an empirical hypothesis H which it doesn't entail. This relation between E and H – sometimes called 'partial entailment' – can't be a *chance* because it's not *empirical*. That's because any relevant empirical *data* about chances will – by definition – be in the evidence, E, while any relevant *hypotheses* about them will be in H. So, while chances can be among the *relata* of this relation, they can't *be* the relation, which is therefore knowable, if at all, only *a priori*.

4. The fact that inductive probability isn't empirical is also what stops it being incompatible with *determinism*, which I take to be the *empirical* thesis that, roughly speaking, everything in time has a sufficient cause. The reason inductive probabilities can't be incompatible with *that* is that they can't require their empirical *relata* to include any facts or hypotheses, about chances or anything else, which contradict – or, come to that, entail – determinism.

5. The *other* probabilities I need to distinguish from chances are *credences*, which some decision theories (e.g. Ramsey 1926) postulate as probability measures of *degrees of belief*. Credences aren't *inductive* probabilities for the same reason chances aren't – they're *empirical*: our credences are empirical properties of *us*. Hypotheses about them are supported by evidence

about what our *desires* make us *do*, while they in turn support hypotheses about what our desires *would* – or, on some readings of decision theory, *should* – make us do in other situations. So, while a credence, like a chance, can be a *relatum* of an inductive probability relation, it too can't be the relation itself.

6. But nor are credences *chances*. A credence of 0.5 in a coin toss landing heads isn't a 0.5 *chance* of *believing* it'll land heads: it's a distinct state of *doubt*, half-way between belief and disbelief. Nor is it *believing* there's a 0.5 *chance* of heads, since credences needn't correspond to beliefs in chances at all, let alone *be* them. If you know that a tossed coin is either double-headed or double-tailed, but not *which*, your credence in its landing heads may well be 0.5, even though you believe its *chance* of doing so is either zero or nearly 1.

7. Nor, by the way, do the decision theories that postulate credences *also* postulate *chances*, because they're *deterministic*: they say, rightly or wrongly, what our desires and credences *will* (or *should*) make us do, not what our *chances* of doing so will be or should be. Credences are just as compatible with determinism as inductive probabilities.

8. In short, chances, inductive probabilities and credences are quite different applications of mathematical probability; just as *wave* theories of light, sound and water are of wave *equations*, like the one which says that a wave's speed is its frequency times its wavelength. Satisfying *those* equations, which is what makes us *call* things 'waves', is *all* that light, sound and water waves have in common. It *doesn't* make them the same kind of thing, and no one thinks it does. No one expects a theory of *light* waves to apply to sound waves, and rejects it because it doesn't: no one expects *any* theory of what waves *are* to be true of all of them.

9. Equally, the fact that chances, inductive probabilities and credences all apply the measure that makes us *call* them 'probabilities' *doesn't* make them a single kind of thing, of which a single theory of what they are might be true; and they're not. Limiting-frequency and single-case theories of chance don't apply to *credences*; functionalist theories of credence don't apply to *chances*; and *none* of the above apply to inductive probabilities. So it's no objection to *any* of these theories that *none* of them applies to probabilities of all three kinds. No non-mathematical theory *could*, and no one should reject *any* such theory because it doesn't.

10. There is *one* excuse for holding a single theory of *all* probabilities, that's provided by a *dis*analogy with waves. For while no one denies the *phenomena* of light, sound and

fluctuations in liquid surfaces that wave theories of them *explain*, that's *not* true of these three kinds of probability. Some philosophers don't believe in *credences*, because they're fairly sure that beliefs don't come by *degrees*. Others, including me, don't believe in *a priori inductive* probabilities, because we don't think *entailment* comes by degrees (Ramsey 1926 p. 57).

11. Chance is more complicated, because chances are invoked to explain certain frequencies, like those of male and female births in large populations, and as *all* frequencies have a *probability* measure, no one can deny those *statistical* probabilities. What *can* be denied is that *any* of these frequencies are explained by probabilities that *aren't* just other frequencies, which is why *that* from now on is what I'm going to *mean* by 'chances'.

12. With that caveat, *any* of our three kinds of probability *can* be denied, so if only *one* kind is accepted, there *can* be a single true theory of *that*. Those of us who accept *more* than one kind – in my case *chances* and *credences* – have more work to do, and not only because we need more than one theory of probability. The other reason is that *our* theories, unlike wave theories, may need to be *linked*. For while nothing about any *one* kind of wave implies *anything* about the other kinds, that may *not* be true of our three kinds of probability.

13. Take for example what I call the 'chances-as-evidence' or 'C-E' principle (Mellor 2005 ch. 6.IV). This, in the example in **Note 2**, says that if *all* you know about how a coin toss will land is that its *chance* of landing heads is p, then your *credence* that it *will* land heads should *also* be p. So anyone who, like me, *accepts* this principle will need to make their theories of chance and credence *consistent* with it – which, as we'll see later, isn't a trivial requirement.

14. Meanwhile, however, having *noted* the principle, we can ignore it for the time being, because it *presupposes* that chances and credences are different *kinds* of probability, one of which – credences – I've already argued is compatible with determinism. And whether the *other* kind, chances, are *also* compatible with determinism depends only what chances *are*, whatever their normative implications for credences. So: what *are* chances?

2 Chance, determinism and conditionals

15. To my mind the only tenable theories of chance are variants of the two I list in **Note 3**. The first is, as its name suggests, a *sort* of *frequency* theory; but it doesn't say that chances *are* the frequencies they explain, and not only because nothing explains itself, but because not *all*

chances *match* those frequencies. If a coin with a 0.5 chance of landing heads when tossed in a certain way is only tossed that way once – or indeed *any odd* number of times – the frequency of heads *can't* be *exactly* 0.5. But if such tosses were repeated *endlessly*, that frequency could *tend* towards 0.5; and that *limit*, if it exists, is what a 'limiting frequency' theory of chance (e.g. von Mises 1957) says that a 0.5 chance of heads *is*.

16. Now this theory *can't* make chances incompatible with determinism. No *frequency* of heads in a class of actual tosses, let alone in a class of merely *hypothetical* ones, can stop something making each actual toss land whichever way it *does* land. Only a so-called 'single-case' theory, on which a coin toss's chance of landing heads is a property *of that very toss*, could do that. And as that's a theory I accept anyway on other grounds, which I shan't go into unless you ask me to, it's the one that from now on I'm going to take for granted, mainly because I want to draw out some of its more interesting – and perhaps surprising – implications.

17. On a single-case theory, then, a coin toss's chance *p* of landing heads is a property of *that toss* such that the *frequencies* of heads in larger and larger classes of tosses *with that property* have a limiting frequency *p*. And similarly in other cases. So the question is: are chances, so understood, compatible with determinism? Can a coin toss with this chance property, [capital] *P*, *also* have a property *Q* which makes *all* and *only* tosses that are *Q* land heads?

18. Perhaps the easiest way to see that it *can* is to compare *P* with properties, like solubility, that are called 'dispositions' because they're identified by the conditionals, like 'if it's put in water it'll dissolve', which, in the circumstances, they make true.

19. (Two points of terminology here, before I go on. The first is that if you don't think conditionals *have* truth-values, you can read 'true' as 'truth-*preserving*' – meaning the consequent will be true if the antecedent is – and 'false' as '*not* truth-preserving': it won't affect the argument. Secondly, if you don't believe in truthmakers, you can replace 'is made true by' with one of three well-known euphemisms: 'is true in virtue of', 'is grounded in' or 'is realised by'. that too won't affect the ensuing argument.)

20. Then the first thing to say about solubility is this. A 1-gram thing *a* with a water-solubility *S* of 1 gm per *N* litres may nevertheless *not* dissolve in *N* litres, because *putting* it in water may *reduce* its solubility. What *a*'s solubility *S* makes true *isn't* 'if *a*'s put in *N* litres of water it'll

dissolve' but 'if *a*'s put in *N* litres of water *and is still S* it'll dissolve': that's the conditional C in **Note 4**, which I follow Carnap (1936–7 §5) in calling a '*reduction sentence*'.

21. So *a* will be water-*soluble* at a time *t* if it *then* has a property which, whatever *else* it does, makes C *true* at *t* for some *N*; and *a* will be water-*in*soluble at *t* if it then has a property which, among other things, makes C *false* at *t* for *all N*. And that's why, while things can *become* more or less soluble, nothing can have *more* than one solubility at a time – because for no *N* can C be both true and false of the same thing at the same time.

22. Now take *velocity*. A train going a mile a minute may *not* be a mile away a minute later, because it may speed up or slow down in that time. So the conditional its speed makes true *isn't* 'if it's a minute later it'll be a mile away' but the *reduction sentence* 'if it's a minute later *and its speed hasn't changed* it'll be a mile away'. Here too, what stops a train going at more than one speed at once is the fact that a reduction sentence like *that* can't be both true and false of the same train at the same time.

23. But this *doesn't* mean the train can't *speed up*. As well as the property, its *speed*, which if it persists for a minute, will move it a *mile*, it can *also* have a property, an *acceleration*, which if *it* persists for a minute, will move it *more* than a mile. For while that acceleration will falsify the simple conditional 'if it's a minute later it'll be a mile away', it *won't* falsify the reduction sentence 'if it's a minute later *and its speed hasn't changed* it'll be a mile away'.

24. And so it is with chance and determinism. A coin toss's chance *p* of landing heads makes true the reduction sentence 'if the toss is one of *N* tosses *with the same chance of landing heads*, then the limiting frequency of heads, as *N* increases indefinitely, will be *p*'. This no more stops a toss which *does* land heads *also* having a property *Q* which makes all and only tosses *that are Q* land heads than a train's speed stops it accelerating.

25. This is what makes single-case chances compatible with determinism: a coin toss having, say, a 0.5 chance of landing heads can't stop a property *Q* making that toss land heads. Indeed, since it can't stop it landing heads even if *no* such property *Q exists*, it can't falsify the simple conditional 'if the coin is tossed it'll land heads' anyway. All it can do, if this chance is all you *know* about the coin toss, is stop you *knowing in advance* whether that conditional is true.

26. This needn't, by the way, make *every* chance of heads less than 1 stop you knowing in advance that a coin toss will land heads. *That* depends on what it takes to *know* something,

which is, as you know, a contentious matter. But there is *one* view of it which, as a *reliabilist* about knowledge, I favour but shan't argue for, on which knowing that a coin toss's chance of heads is *very close* to 1 *can* amount to knowing in advance that it *will* land heads.

27. Suppose that all I know about how a coin toss will land is that its *chance* of landing heads is, say, 0.999. Then my C-E principle says that my *credence* in that outcome should *also* be 0.999. But that's too close to 1 to affect what, in most situations, a normative decision theory will tell me to *do*: namely, in effect, to bet on heads. For it will only tell me *not* to do that if a £1 bet *against* heads would get me more than £1000 if I *won*. So if how the coin lands matters *less* to me than that, as it usually will, then for reasons given in David Lewis's 1996 paper 'Elusive knowledge', I think that, if the coin *does* land heads, then my prior 0.999 *credence* in its landing heads, derived from that chance, *will* count as *knowing* it will land heads. In short, I think you *can* know something if you'll win what you know to be a good enough bet on it.

3 Measuring chances

28. Well, so far, *perhaps* so good; but hardly far enough, since my C-E principle applies only to chances that are *known*. But how can we *know* what a coin toss's chance of landing heads *is*, if its value is fixed by something no one can ever *observe*, namely a limiting frequency? How can we tell (a) that the relevant limiting frequency *exists* and (b) what, if it does, its *value* is?

29. That these questions *have* answers isn't undeniable, for since some frequencies clearly *aren't* explained by chances – the one-in-four frequency of leap years in any century for example! – maybe *none* are. *All* the chances postulated by scientific theories *can* be denied, as for example those of human evolution are by creationists, and those of man-made global warming are by climate-change sceptics. But the question *here* is *why*, religious and short-sighted commercial motivations apart, *would* anyone deny the existence of chances postulated by these or other generally accepted scientific theories.

30. I can think of three reasons, all bad. One is that chances are incompatible with determinism, which, as we've seen, they're not. Another is the general underdetermination of theory by data, which applies no more to chances than to any other theoretical entity. The third, and least bad, is that chances *can't* explain frequencies, because they're never either *sufficient* or *necessary* for them, which indeed they aren't. A coin toss's chance p of landing heads doesn't

entail *any actual* frequency of heads: all it entails is that, in a class of *N* tosses, that frequency's *chance* of being *close* to *p* gets nearer and nearer to 1 as *N* increases.

31. But if explanations *had* to be sufficient and/or necessary for what they explain, *none* of the frequencies that chances are invoked to explain would have *any* non-trivial explanation, and no one here, I hope, believes *that*. Hence of course the development and defence of theories of explanation that *weaken* this requirement in various ways. One such theory, for example, which I accept, for reasons I shan't go into, only requires *causal* explanations to raise the *chances* of what they explain: which is how, for example, smoking explains the higher incidence of lung cancer in smokers than in non-smokers.

32. Still, none of this answers the question of how to *detect* and *measure* chances. How is *that* done? Well, the *prima facie* evidence for chances is frequencies which, unlike those of leap years, vary a lot in small classes of a certain kind but less and less as the classes get larger – as, I'm supposing, those of heads do in classes of coin tosses of some kind *W*. But then, in order to explain these frequencies by single-case *chances*, we must know how to tell that two tosses of kind *W* have the *same* chance of landing heads, *before* we know what that chance *is*.

33. Now this question doesn't arise only for *chances*. Suppose for example we want to use a thermometer to measure an air temperature. Since the thermometer can't *do* that until it's *at* the temperature we want to measure, we must *get* it to that temperature *before* it can tell what that temperature is. What tells us how to do *that*?

34. The answer is of course, as I say in **Note 5**, that a theory we accept tells us what a thermometer's temperature is a *function* of: in this case, not *only* of the air temperature we want to measure, but *also* of incident radiation and wind speed. *That's* what tells us to put the thermometer in the shade, and out of the wind, before we ascribe its temperature to the air.

35. And as with air temperatures, so with chances. It's why theories that postulate chances need to say what they take those chances – of unstable atoms decaying, species surviving, people inheriting disorders, catching flu, getting cancer, and so on – to be *functions* of. *That's* what enables them to tell us when two events have the *same* chance of whatever it is, and therefore *which* frequencies, in *which* classes, can be used to *measure* that chance.

36. By the way, a function's *values* mustn't be confused with its *arguments*. Take the theory that a gas's absolute temperature is proportional to a function – the mean – of its molecules' kinetic

energies. This function's *arguments* are kinetic energies, but its *values* aren't: they're *temperatures*. Being a *function* of kinetic energies doesn't make *temperatures* kinetic energies, even in gases, let alone in solids, liquids and black-body radiation, to which the theory doesn't apply. Similarly, just because theories that postulate chances tell us what their values are *functions* of, this doesn't mean they tell us what *chance* is, either in *their* subject matters or in any other.

37. Moreover, what theories *do* tell us, about what temperatures or chances *depend* on, still won't tell us how to *measure* them. Even if you know what a temperature or a chance is a function *of*, you still have to *measure* it in order to discover what the function *is*: e.g. *how* hot a gas is when its molecules have a certain mean kinetic energy; or *what* chance an atom of radium-226 has of decaying in the next century.

38. So even when we know *which* frequencies, in *which* classes, a chance explains, we still have to infer the unobservable *limiting* frequency that's its value from actual frequencies that we *do* observe. And with *that* inference, my chances-as-evidence principle is of course no help at all, since these chances aren't *evidence*: they're what we need evidence *for*.

39. Still, there *are* theories about how to infer limiting frequencies from actual ones, though they are rather contentious. So it's fortunate for me that I needn't discuss them, since they depend more on theories of *evidence* than on theories of *chance*. All I *will* say, off the record, is that *here*, if not elsewhere, I'm *inclined* to favour inference to the best explanation since, as I say in **Note 6**, that reduces in *this* case to the *maximum likelihood* principle, which tells us to infer the range of chances that gives the observed frequencies their highest chances of occurring.

4 Chance, propensity and intensionality

40. Having said all I have to say about how to *measure* single-case chances, I want to finish by saying a bit more about what *kind* of properties they are. In my 2005 book on probability, I, like many authors, call them '*propensities*', after a 1957 paper by Karl Popper. But a forthcoming paper by Mauricio Suárez has persuaded me that the term 'propensity' would be *better* reserved for a *derivative* property, two examples of which I give in **Note 7**.

41. Take my first example: a coin's *bias*, or lack of it. A *un*biassed coin, *a*, *isn't* one with a 0.5 chance of landing heads: *a*'s chance of landing heads is *zero* unless it's being tossed, and even

then may only be 0.5 if it's tossed in some understood way *W*. So *a*'s *lack* of bias isn't a 0.5 *chance* of landing heads: it's a deterministic *disposition* to *have* that chance if it's tossed in a way *W* which doesn't bias it. In other words, it's a property of *a* – call it *F* (for 'fair') – which makes true the reduction sentence 'if *a* is tossed in way *W* and *is still F*, its chance of landing heads will be 0.5'. *That's* the kind of property I now want to call a 'propensity'.

42. My second, more serious, example is a radioactive atom's *instability*. By this I mean the property that's measured by its *half-life*, the time in which its chance of decaying is 0.5, which for an atom of radium-226 is about 1600 years. But this property isn't *that* chance – or any *other* of the atom's myriad chances of decaying in different times: within a month, a year, a century, etc. It's a *propensity*: the property that disposes the atom to *have* all these chances – provided of course the property doesn't *change* before the atom decays; as it would, for example, if the atom was bombarded.

43. So an atom's propensity to decay is a *changeable property* of it, just as its velocity is. Indeed the reduction sentences each makes true share the same *antecedent* – 'if it's *N* units of time later and the property hasn't changed' – and differ only in their *consequents*: one giving its *chance of decaying*, and the other saying far it will *travel*, in that time.

44. If propensities are properties of things, like coins and atoms, *chances*, on the face of it, are properties of events. But *which* events? What event is a coin toss's chance of landing heads a property of? It can't be a property of the coin's landing heads, since that event may not *occur*. Nor, for the same reason, can it be a *relation* between that event and the coin toss. The only event it *can* be a property of is, as I've said, and say again in **Note 8**, the toss itself.

45. The question then is: what makes this property a chance of the tossed coin *landing heads*, and not, for example, of its spinning ten times before it lands? The answer *is* of course the fact that the *frequencies* whose limit is this chance's *value* are those of tossed coins *landing heads*, *not* those of coins spinning ten times before they land. *That's* what fixes what this chance is a chance *of*, namely *landing heads* – something which, since it's what a *credence* that the coin will land heads is a credence *in*, I'm going to call the chance's '*content*'.

46. That chances have contents, in the same sense that credences do, is in fact *presupposed* by *any* principle which, like my C-E principle, says that credences should sometimes *equal* the

corresponding chances. And they can only do *that* if anything there's a *chance of* is something there can be a *credence in*.

47. The fact that chances *have* contents has another interesting implication, namely that, as I say in **Note 9**, chances are as *intensional* (with an 's') as credences are. For one thing, they too can be about entities, like the event of a coin landing heads, that may not exist; and, for another, their values can also depend on how any *real* entities they *are* about are referred to. Last October, for example, the fact that my credence in Donald Trump's being the 45th US President, which he *is*, was less than 1, didn't make my credence in *his being Donald Trump* less than 1. And nor, alas, could his then less-than-1 *chance* of being the 45th President have given him a less-than-1 *chance* of being Donald Trump.

48. The fact that chances are as intensional as credences doesn't of course show that credences are *physical*, or that chances are *mental*: they're not. Nor does it show that credences are *reducible* to chances, or *vice versa*: they aren't. All it shows is that intensionality isn't the preserve of the mental. On the contrary: it's far more common in the *inanimate* world, if only because every radioactive atom in the universe has, at any time, a propensity to decay which entails infinitely many chances of its decaying in different time intervals.

49. There is of course a lot more to be said about chances, for example, about whether, say, a coin toss's chance of landing heads measures how *possible* (in a *non*-epistemic sense) that outcome is. But I suspect that what I've already said may be more than enough to go on with, partly because it suggests some interesting but debatable *general* inferences. For example: that there are more things in heaven and earth than are dreamt of in an *extensionalist* philosophy; that causation isn't a relation between *events* in the sense of Donald Davidson's 1970 article 'Events as particulars'; and that there isn't, and never was, any problem of mental causation.

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